Int. J. Multiphase Flow Vol. 12, No. 5, pp. 745–758, 1986
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0301-9322/86 $3.00 + 0.00

LOCAL INSTANT FORMULATION OF TWO-PHASE FLOW

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(*Received* 22 *April* 1985; *in revised form* 27 *November* 1985)

Abstract—The local instant formulation of mass, momentum and energy conservations of two-phase flow has been developed. Distribution, an extended notion of a function, has been introduced for this purpose because physical parameters of two-phase flow media change discontinuously at the interface and the Lebesgue measure of an interface is zero. Using a characteristic function of each phase, the physical parameters of two-phase flow have been defined as field quantities. In addition to this, the source terms at the interface are defined in terms of the local instant interfacial area concentration. Based on these field quantities, the local instant field equations of mass, momentum and total energy conservations of two-phase flow have been derived. Modification of these field equations gives the single field representation of the local instant field equations of two-phase flow. Neglecting the interfacial force and energy, this formulation coincides with the field equations of single-phase flow, except in the definition of differentiation. The local instant two-fluid formulation of two-phase flow has also been derived. This formulation consists of six local instant field equations of mass, momentum and total energy conservations of both phases. Interfacial mass, momentum and energy transfer terms appear in these equations, which are expressed in terms of the local instant interfacial area concentration.

1. INTRODUCTION

Two-phase flow phenomena are of extreme importance in various fields of science and technology such as geophysics, nuclear engineering, chemical engineering etc. In analyzing two-phase flow phenomena, one needs a set of basic equations which describe the conservations of mass, momentum and energy of two-phase flow media.

For single-phase flow, such basic equations are rigorously given in the form of mass, momentum and energy balances in infinitesimal volume, dv, and infinitesimal time duration, dt. These equations constitute the local instant field equations for density, velocity and energy, which can be applied to all the volume and time domains under consideration.

However, for two-phase flow, such local instant field equations have not been attainable without adopting appropriate averaging and/or modeling. Traditionally, the earliest model of two-phase flow is the homogeneous mixture model. In this model, both phases are assumed to be completely mixed and move with the same speed. Based on these assumptions, the density, velocity and energy of the two-phase mixture are defined and the local instant field equations of mass, momentum and energy conservations obtained.

Zuber & Findley (1965), Wallis (1969), Ishii (1977) and Ishii & Zuber (1979) have taken into consideration the diffusion effects of each phase, which mean that the two-phase move with different speeds. This model is called the "drift flux model" and has been widely used in two-phase flow analyses.

Truesdel *et al.* (1984) have applied rational thermodynamics to the mixture model of two-phase flow and obtained useful results for fluid suspensions etc. Although the mixture model has formulated local instant field equations of two-phase flow, it has limitations due to the assumptions on which it is based. In this model, it is assumed that in any small volume of mixture, both phases coexist. This assumption is acceptable when one phase is finely dispersed in the other, such as the suspension of fine particles in fluid etc. However, when the size of the dispersed phase increases appreciably, this model cannot help showing some deviations from reality.

Another direction of two-phase flow formulation has been pursued by Ishii (1975), Delhaye (1968), Delhaye *et al.* (1981), Bouré (1973), Wallis (1969) and Kocamustafaogullari (1971), Banerjee & Chan (1980) and Banerjee (1980) among others.

They have developed the two-fluid model (or separated flow model) formulation of two-phase flow. In this model, each phase is treated separately and the interface is regarded as a moving boundary. In each phase, the local instant conservation equations in single-phase flow can be written down. In addition, at the interface, local instant balances of mass, momentum and energy are formulated as boundary conditions. The above-mentioned equations are then averaged over time and space domains. Using several integral theorems (Leibniz's rule, Gauss's theorem etc.) the averaged field equations of density, velocity and energy are derived, respectively, for each phase. This formulation accurately reflects the physical aspects of two-phase flow. However, the field equations obtained are given in averaged forms in a certain volume or over a certain time period. Local instant balances of mass, momentum and energy in two-phase flow are not represented in the form of field equations. It should be noted that the local instant formulations used in the two-fluid model described above are valid only in each phase or at the interface, they are not the local instant field equations which are valid for all the space and time domains under consideration.

In view of the above, it is highly desirable that the local instant field equations of mass, momentum and energy conservation of two-phase flow are formulated without any averaging and/or modeling. Based on this formulation, various averaging procedures of the basic equations can be done easily without using complicated integral theorems. Furthermore, it facilitates statistical averaging of the basic equations of two-phase flow. Some statistical treatments have been already applied to the local interfacial area concentration (Kataoka *et al.* 1984). Recent developments in measurement techniques for two-phase flow (laser anemometry etc.) have provided detailed knowledge of the microscopic structures of two-phase flow. Based on these data and the present formulation, statistical treatment of two-phase flow is now possible. Such a formulation will be particularly useful in analyzing the microscopic structures of two-phase flow (turbulence etc.) and in evaluating the applicability of various two-phase flow models.

In two-phase flow fields, density, velocity and energy change discontinuously at the interface. Therefore, when one considers the local instant balances of mass, momentum and energy, one inevitably faces the differentiation of the discontinuous functions which represent the density, velocity and energy fields. Such differentiation cannot be executed in the ordinary notion of a function (Schwartz 1950, 1961).

Furthermore, in two-phase flow fields, interfaces play an important role in mass, momentum and energy transfer. Therefore, source term functions must be defined at the interface. However, since the Lebesgue measure of an interface is zero in three-dimensional space, again such source term functions cannot be represented by the ordinary notion of a function (Schwartz 1950, 1961).

These difficulties in mathematical treatment are considered to be the main reason why the local instant field equations of two-phase flow have not been obtained so far.

The difficulties mentioned above can be overcome by introducing "distribution", the theory of which has been mathematically established by Schwartz (1950, 1961). It is an extended notion of a function and, in the meaning of "distribution", the differentiation of a discontinuous function is possible in any higher order derivative. Furthermore, the source term function at zero Lebesgue measure can be represented in terms of distribution.

Traditionally, one example of distribution was previously invented by Dirac (1958) and has been widely used in physics and engineering without mathematical verification. In averaging theory of two-phase flow, this distribution has been conveniently used by Gray & Lee (1977).

Recently, Kataoka *et al.* (1984) have derived the local instant formulation of the interfacial area concentration of two-phase flow in terms of distribution. This local instant interfacial area concentration is indispensable when considering the local instant balances of mass, momentum and energy in two-phase flow.

In this paper, using the local instant interfacial area concentration and differentiation of discontinuous functions in the meaning of distribution, the local instant field equations of mass, momentum and energy conservation in two-phase flow are derived.

2. LOCAL INSTANT REPRESENTATION OF THE PHYSICAL PARAMETERS IN TWO-PHASE FLOW

In order to formulate the local instant field equations of mass, momentum and energy conservation of two-phase flow, it is necessary to represent the physical parameters of two-phase flow, such as density, velocity, internal energy etc. as field quantities. This means that these physical parameters must be expressed by appropriate functions which are defined in all the space and time domains under consideration.

For this purpose, the characteristic function of each phase will be introduced here. One defines the equation which represents the interface by

$$f(x, y, z, t) = 0. \qquad [1]$$

Here, the interface can be a single continuous surface or a group of closed surfaces which, for example, reflect bubbles in gas–liquid two-phase flow.

Using [1], the regions where phases 1 and 2 exist can be defined by following relations:

$$\text{for phase } 1, f(x, y, z, t) > 0; \qquad [2]$$

and

$$\text{for phase } 2, f(x, y, z, t) < 0. \qquad [3]$$

Then the characteristic function of each phase, $\phi_k(x, y, z, t)$ $(k = 1, 2)$ can be introduced by

$$\phi_1(x, y, z, t) = h[f(x, y, z, t)]$$
$$\text{(characteristic function for phase 1)} \qquad [4]$$

and

$$\phi_2(x, y, z, t) = 1 - h[f(x, y, z, t)]$$
$$\text{(characteristic function for phase 2).} \qquad [5]$$

Here, $h(w)$ is the Heaviside function which is defined by

$$h(w) = 0 \quad (w < 0) = 1 \quad (w > 0). \qquad [6]$$

For example, when phases 1 and 2 represent gas and liquid phases, respectively, $\phi_1(x, y, z, t)$ can be called the "local instant void fraction" which takes a value of zero or unity. The space or time-averaged value of ϕ_1 corresponds to the "void fraction" which is usually used in two-phase flow analysis. The averaged void fraction takes a continuous value between zero and unity.

Now, for each phase $(k = 1, 2)$, one denotes density by ρ_k, velocity by $\mathbf{v}_k$, internal energy per unit mass by U_k, pressure by P_k, the stress tensor by τ_k, external force per unit mass by $\mathbf{F}_k$, the heat flux vector by $\mathbf{q}_k$ and the external heat source by Q_k. These quantities are defined only in each phase (region for $\phi_k = 1$). Therefore, they are not the field quantities which are defined in all the space and time domains under consideration. One then multiplies these quantities by the characteristic function of each phase:

$$\phi_k\rho_k,\ \phi_k\mathbf{v}_k,\ \phi_k U_k,\ \phi_k P_k,\ \phi_k\tau_k,\ \phi_k\mathbf{F}_k,\ \phi_k\mathbf{q}_k,\ \phi_k Q_k.$$

These quantities can be regarded as the field quantities. In this way, the physical parameters in two-phase flow are represented locally and instantaneously as the field quantities.

One should note the following relations concerning the characteristic functions:

$$(\phi_k)^n = \phi_k \quad (k = 1, 2) \qquad [7]$$

and

$$\phi_k\phi_j = 0 \quad (k \neq j;\ k, j = 1, 2). \qquad [8]$$

These relations are evident from [4]–[6]. Using [7] and [8], the product of any two of the field quantities, $\phi_k A_k$ and $\phi_k B_k$, can be given by

$$(\phi_k A_k)(\phi_k B_k) = \phi_k A_k B_k. \qquad [9]$$

Here the product operator includes scalar, vector and tensor products.

Besides the field quantities of two-phase flow described above, there are other important field quantities concerning the interface. For this purpose, the local instant interfacial area, $a_i(x, y, z, t)$, can be conveniently used. The local instant formulation of the interfacial area concentration has been obtained already by Kataoka *et al.* (1984) in terms of distribution since the Lebesgue measure of an interface in three-dimensional space is zero; it is given by

$$a_i(x, y, z, t) = |\mathrm{grad} f|\,\delta(f(x, y, z, t)), \qquad [10]$$

where

$$|\mathrm{grad} f| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2}. \qquad [11]$$

Here, $\delta(w)$ is the distribution, usually called "the δ-function" (Dirac 1958; Schwartz 1950, 1961) and defined by

$$\int_{-\infty}^{\infty} \delta(w - a)\psi(w)\mathrm{d}w = \psi(a). \qquad [12]$$

Here, $\psi(w)$ is an arbitrary function which is continuous at $w = a$. The δ-function is related to the Heaviside function by (Schwartz 1950, 1961)

$$\frac{\mathrm{d}}{\mathrm{d}w} h(w) = \delta(w). \qquad [13]$$

Here, differentiation is in the meaning of distribution, which is defined by Schwartz (1950, 1961). In relation to the interface, one denotes interfacial energy per unit interfacial area by U_s, interfacial force per unit interfacial area by $\mathbf{F}_s$, interfacial area generation rate per unit interfacial area by Γ_s and velocity of the interface by $\mathbf{v}_i$.

3. LOCAL INSTANT FIELD EQUATIONS OF TWO-PHASE FLOW

Based on the physical parameters of two-phase flow, defined as the field quantities in section 2, one can formulate the local instant field equations of mass, momentum and energy conservation of two-phase flow.

Firstly, the conservation of mass will be considered. Since the mass of the interface can be regarded as zero, the density of the two-phase media can be given by

$$\sum_{k=1}^{2} (\phi_k \rho_k).$$

When there is no net mass generation in the two-phase media as a whole, the local instant field equation of the mass conservation of two-phase flow is given by

$$\frac{\partial}{\partial t}\left[\sum_{k=1}^{2} (\phi_k \rho_k)\right] + \mathrm{div}\left[\sum_{k=1}^{2} (\phi_k \rho_k \mathbf{v}_k)\right] = 0. \qquad [14]$$

Here, differentiation is in the meaning of distribution (Schwartz 1950, 1961). Equation [14] is the field equation which determines the density field of two-phase flow. It describes the mass conservation of two-phase media at any time and in any position, i.e. in phases 1 and 2 and at the interfaces. This can be confirmed in the following manner.

Equation [14] can be rewritten as

$$\sum_{k=1}^{2} \left\{\phi_k \left[\frac{\partial \rho_k}{\partial t} + \mathrm{div}(\rho_k \mathbf{v}_k)\right]\right\} + \sum_{k=1}^{2} \left(\rho_k \frac{\partial \phi_k}{\partial t} + \rho_k \mathbf{v}_k \cdot \mathrm{grad}\, \phi_k\right) = 0, \qquad [15]$$

where · means the scalar product of two vectors. In each phase, or in the region where

$f(x, y, z, t) \neq 0$, [15] becomes

$$\frac{\partial \rho_k}{\partial t} + \operatorname{div}(\rho_k \mathbf{v}_k) = 0 \quad (k = 1, 2). \tag{16}$$

Equation [16] is consistent with the fact that within each phase, the single-phase mass conservation equation must be satisfied.

On the other hand, at the interface or in the region where $f(x, y, z, t) = 0$, [15] becomes, in view of [4]–[6] and [13],

$$\sum_{k=1}^{2} \left[\rho_{ki} \frac{\partial f}{\partial t} \delta(f) + (\rho_{ki} \mathbf{v}_{ki} \cdot \operatorname{grad} f) \delta(f) \right] = 0, \tag{17}$$

where, ρ_{ki} and $\mathbf{v}_{ki}$ are the values of ρ_k and $\mathbf{v}_k$ at the interface; defined by

$$\rho_{1i} = \lim_{f(x,y,z,t) \to +0} \rho_1, \tag{18}$$

$$\mathbf{v}_{1i} = \lim_{f(x,y,z,t) \to +0} \mathbf{v}_1, \tag{19}$$

$$\rho_{2i} = \lim_{f(x,y,z,t) \to -0} \rho_2 \tag{20}$$

and

$$\mathbf{v}_{2i} = \lim_{f(x,y,z,t) \to -0} \mathbf{v}_2. \tag{21}$$

Geometrical consideration of a moving surface shows that

$$-\frac{\partial f}{\partial t} = \mathbf{v}_i \cdot \operatorname{grad} f, \tag{22}$$

$$\mathbf{n}_{1i} = -\frac{\operatorname{grad} f}{|\operatorname{grad} f|} \tag{23}$$

and

$$\mathbf{n}_{2i} = \frac{\operatorname{grad} f}{|\operatorname{grad} f|}. \tag{24}$$

Here, $\mathbf{n}_{ki} (k = 1, 2)$ are the outward unit normal vectors of each phase, as shown in figure 1.

Substituting [22]–[24] into [17], and in view of [12], one obtains

$$\sum_{k=1}^{2} [-\rho_{ki}(\mathbf{v}_{ki} - \mathbf{v}_i) \cdot \mathbf{n}_{ki}] = 0. \tag{25}$$

In [25], the value $-\rho_{ki}(\mathbf{v}_{ki} - \mathbf{v}_i) \cdot \mathbf{n}_{ki}$ represents the mass flux of phase k at the interface. Since there is no mass generation at the interface, the sum of the mass fluxes of both phases must be zero. Therefore, [25] indicates the mass balance at the interface. This relation has been derived by Scriven (1960), Standard (1964), Ishii (1975), Delhaye (1974) and others from integral theorem, and is sometimes called the "local instant mass jump condition".

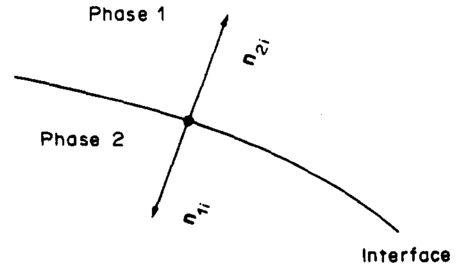

Fig. 1. Outward unit normal vectors of each phase at the interface.

Thus, as shown in [16] and [25], it is confirmed that [14] satisfies two-phase mass conservation in both phases and at the interface. Therefore, the local instant field equation of mass conservation of two-phase flow can be represented by [14].

It should be noted that in [17], the term $\delta(f)$ appears as a result of the differentiation of the characteristic function in the meaning of distribution. As shown in [10], this term is related to the local instant interfacial area concentration, derived previously by Kataoka *et al.* (1984). Equation [10] is originally derived from geometrical consideration of the interface. Here, the interfacial area concentration is consistently derived from the local instant field equation of mass conservation.

In the following, the conservation of momentum of two-phase flow will be described. Since the momentum of the interface is considered to be zero, the momentum of the two-phase media per unit volume is given by

$$\sum_{k=1}^{2} (\phi_k \rho_k \mathbf{v}_k).$$

On the other hand, the force applied to a unit volume of the two-phase media is given by

$$-\operatorname{grad}\left[\sum_{k=1}^{2} (\phi_k P_k)\right] + \operatorname{div}\left[\sum_{k=1}^{2} (\phi_k \tau_k)\right] + \sum_{k=1}^{2} (\phi_k \rho_k \mathbf{F}_k) + \mathbf{F}_s a_i.$$

Here, $\mathbf{F}_s$ represents the surface tension force per unit interfacial area; as will be shown later, it is balanced by the difference of pressure, shear stress and momentum flux between the phases. Since a_i is defined as the interfacial area per unit volume, the product $\mathbf{F}_s a_i$ represents the surface tension force acting on a unit volume of two-phase media containing the interface. Therefore, the local instant field equation of conservation of momentum of two-phase flow is given by

$$\frac{\partial}{\partial t}\left[\sum_{k=1}^{2} (\phi_k \rho_k \mathbf{v}_k)\right] + \operatorname{div}\left[\sum_{k=1}^{2} (\phi_k \rho_k \mathbf{v}_k \mathbf{v}_k)\right] = -\operatorname{grad}\left[\sum_{k=1}^{2} (\phi_k P_k)\right] + \operatorname{div}\left[\sum_{k=1}^{2} (\phi_k \tau_k)\right] + \sum_{k=1}^{2} (\phi_k \rho_k \mathbf{F}_k) + \mathbf{F}_s a_i. \quad [26]$$

Here, the differentiation is in the meaning of distribution (Schwartz 1950, 1961). This is natural because [26] already includes the δ-function as a term of the interfacial area concentration, $a_i = |\operatorname{grad} f|\delta(f)$. The term $\mathbf{v}_k \mathbf{v}_k$ is a dyadic tensor.

Equation [26] is the field equation which determines the velocity field of two-phase flow. It satisfies the momentum conservation of two-phase media at any time and at any location, i.e. within both phases and at the interface. This can be confirmed in detail as shown below.

In a similar way to the mass conservation equation, in view of [4]–[6], [10]–[13] and [22]–[24], [26] can be rewritten as

$$\sum_{k=1}^{2} \left\{\phi_k \left[\frac{\partial}{\partial t}(\rho_k \mathbf{v}_k) + \operatorname{div}(\rho_k \mathbf{v}_k \mathbf{v}_k)\right]\right\} + \sum_{k=1}^{2} [-\rho_{ki} \mathbf{v}_{ki} \cdot (\mathbf{v}_{ki} - \mathbf{v}_i) \mathbf{n}_{ki} a_i] = \sum_{k=1}^{2} [\phi_k(-\operatorname{grad} P_k + \operatorname{div} \tau_k + \rho_k \mathbf{F}_k)] + \sum_{k=1}^{2} [(P_{ki} \mathbf{n}_{ki} - \mathbf{n}_{ki} \cdot \tau_{ki}) a_i] + \mathbf{F}_s a_i. \quad [27]$$

Here, P_{ki} and τ_{ki} are the values of P_k and τ_k at the interface, defined in the same way as [18]–[21].

In each phase, or in the region where $f(x, y, z, t) \neq 0$, [27] becomes

$$\frac{\partial}{\partial t}(\rho_k \mathbf{v}_k) + \operatorname{div}(\rho_k \mathbf{v}_k \mathbf{v}_k) = -\operatorname{grad} P_k + \operatorname{div} \tau_k + \rho_k \mathbf{F}_k \quad (k = 1, 2). \quad [28]$$

This equation is the single-phase momentum conservation equation, which is naturally satisfied within each phase.

On the other hand, at the interface, or in the region where $f(x, y, z, t) = 0$, [27] becomes, in view of [10] and [12],

$$\sum_{k=1}^{2} [\rho_{ki}\mathbf{v}_{ki} \cdot (\mathbf{v}_{ki} - \mathbf{v}_{i})\mathbf{n}_{ki} + \mathbf{n}_{ki}P_{ki} - \mathbf{n}_{ki} \cdot \tau_{ki}] + \mathbf{F}_{s} = 0. \qquad [29]$$

Equation [29] represents the balance of the forces exerted upon the interface, which is consistent with the fact that the interface has no mass and momentum. This relation has been derived by Scriven (1960), Standard (1964), Ishii (1975), Delhaye (1974) and others from integral theorem, and is sometimes called the "local instant momentum jump condition".

From [28] and [29] it has been confirmed that [26] satisfies the momentum balance of two-phase flow within both phases and at the interface. Therefore, [26] is the local instant field equation of two-phase momentum conservation.

Finally, one considers the energy balance of two-phase media. The internal energy, kinetic energy and interfacial energy of two-phase media per unit volume is given by

$$\sum_{k=1}^{2} [\phi_k\rho_k(U_k + \tfrac{1}{2}\mathbf{v}_k^2)] + U_s a_i.$$

On the other hand, the energy generated in and entering the unit volume and the work applied to the unit volume are represented by

$$-\operatorname{div}\left[\sum_{k=1}^{2} (\phi_k\mathbf{q}_k)\right] - \operatorname{div}\left[\sum_{k=1}^{2} (\phi_k P_k\mathbf{v}_k)\right] + \operatorname{div}\left[\sum_{k=1}^{2} (\phi_k\tau_k \cdot \mathbf{v}_k)\right]$$
$$+ \sum_{k=1}^{2} (\phi_k\rho_k\mathbf{F}_k \cdot \mathbf{v}_k) + \sum_{k=1}^{2} (\phi_k Q_k) + (\mathbf{F}_s \cdot \mathbf{v}_i + \Gamma_s U_s)a_i.$$

Then, the local instant field equation of conservation of total energy of two-phase flow can be given by

$$\frac{\partial}{\partial t}\left\{\sum_{k=1}^{2} [\phi_k\rho_k(U_k + \tfrac{1}{2}\mathbf{v}_k^2)]\right\} + \operatorname{div}\left\{\sum_{k=1}^{2} [\phi_k\rho_k(U_k + \tfrac{1}{2}\mathbf{v}_k^2)\mathbf{v}_k]\right\} + \frac{\partial}{\partial t}(U_s a_i) + \operatorname{div}(U_s a_i \mathbf{v}_i)$$
$$= -\operatorname{div}\left[\sum_{k=1}^{2} (\phi_k\mathbf{q}_k)\right] - \operatorname{div}\left[\sum_{k=1}^{2} (\phi_k P_k\mathbf{v}_k)\right] + \operatorname{div}\left[\sum_{k=1}^{2} (\phi_k\tau_k \cdot \mathbf{v}_k)\right]$$
$$+ \sum_{k=1}^{2} (\phi_k\rho_k\mathbf{F}_k \cdot \mathbf{v}_k) + \sum_{k=1}^{2} (\phi_k Q_k) + (\mathbf{F}_s \cdot \mathbf{v}_i + \Gamma_s U_s)a_i. \qquad [30]$$

Here, again, differentiation is in the meaning of distribution (Schwartz 1950, 1961), which is natural since [30] includes the δ-function as the interfacial energy term, $(\mathbf{F}_s \cdot \mathbf{v}_i + \Gamma_s U_s)a_i$. Equation [30] is the field equation which determines the energy field of two-phase flow. It satisfies the total energy conservation of two-phase media at any time and any location under consideration, i.e. within both phases and at the interface. This can be confirmed in the following manner.

Similarly to the mass and momentum conservation equations, in view of [4]–[6], [10]–[13] and [22]–[24], [30] can be rewritten as

$$\sum_{k=1}^{2}\left[\phi_k\left\{\frac{\partial}{\partial t}[\rho_k(U_k + \tfrac{1}{2}\mathbf{v}_k^2)] + \operatorname{div}[\rho_k(U_k + \tfrac{1}{2}\mathbf{v}_k^2)\mathbf{v}_k]\right\}\right] + a_i\left(\frac{\partial U_s}{\partial t} + \mathbf{v}_i \cdot \operatorname{grad} U_s\right)$$
$$+ \sum_{k=1}^{2} [-\rho_{ki}(U_{ki} + \tfrac{1}{2}\mathbf{v}_{ki}^2)(\mathbf{v}_{ki} - \mathbf{v}_i) \cdot \mathbf{n}_{ki}a_i] + U_s\left[\frac{\partial a_i}{\partial t} + \operatorname{div}(a_i\mathbf{v}_i)\right]$$
$$= \sum_{k=1}^{2} \{\phi_k[-\operatorname{div}\mathbf{q}_k - \operatorname{div}(P_k\mathbf{v}_k) + \operatorname{div}(\tau_k \cdot \mathbf{v}_k) + \rho_k\mathbf{F}_k \cdot \mathbf{v}_k + Q_k]\}$$
$$+ \sum_{k=1}^{2} \{[\mathbf{n}_{ki} \cdot \mathbf{q}_{ki} + P_{ki}\mathbf{v}_{ki} \cdot \mathbf{n}_{ki} - \mathbf{n}_{ki} \cdot (\tau_{ki} \cdot \mathbf{v}_{ki})]a_i\} + (\mathbf{F}_s \cdot \mathbf{v}_i + \Gamma_s U_s)a_i. \qquad [31]$$

On the other hand, the conservation of the interfacial area concentration is given by Kataoka (1985):

$$\frac{\partial}{\partial t}(a_i) + \mathrm{div}(a_i \mathbf{v}_i) = \Gamma_s a_i. \qquad [32]$$

Here, U_{ki} and $\mathbf{q}_{ki}$ are the values of U_k and $\mathbf{q}_k$ at the interface, defined in the same way as [18]–[21].

Within each phase, or in the region where $f(x, y, z, t) \neq 0$, [31] reduces to

$$\frac{\partial}{\partial t}[\rho_k(U_k + \tfrac{1}{2}\mathbf{v}_k^2)] + \mathrm{div}[\rho_k(U_k + \tfrac{1}{2}\mathbf{v}_k^2)\mathbf{v}_k] = -\mathrm{div}\,\mathbf{q}_k - \mathrm{div}(P_k \mathbf{v}_k)$$

$$+ \mathrm{div}(\tau_k \cdot \mathbf{v}_k) + \rho_k \mathbf{F}_k \cdot \mathbf{v}_k + Q_k \quad (k = 1, 2). \qquad [33]$$

Equation [33] is the single-phase total energy conservation equation of each phase. It is natural that within each phase, the fluid behaves as a single-phase fluid and obeys the single-phase energy balance law. Therefore, [33] is naturally satisfied within each phase.

On the other hand, at the interface, or in the region where $f(x, y, z, t) = 0$, [31] becomes, in view of [10], [12] and [32],

$$\frac{\partial U_s}{\partial t} + \mathbf{v}_i \cdot \mathrm{grad}\, U_s = \sum_{k=1}^{2} [\rho_{ki}(U_{ki} + \tfrac{1}{2}\mathbf{v}_{ki}^2)(\mathbf{v}_{ki} - \mathbf{v}_i) \cdot \mathbf{n}_{ki} + \mathbf{n}_{ki} \cdot \mathbf{q}_{ki}$$

$$+ P_{ki}\mathbf{n}_{ki} \cdot \mathbf{v}_{ki} - \mathbf{n}_{ki} \cdot (\tau_{ki} \cdot \mathbf{v}_i)] + \mathbf{F}_s \cdot \mathbf{v}_i. \qquad [34]$$

The l.h.s. of [34] represents the increasing rate of interfacial energy per unit interfacial area. On the other hand, the r.h.s. represents the total energy flux by convection, heat flux into the interface and the work exerted on the interface by pressure, stress and interfacial force. Therefore, [34] represents the total energy conservation at the interface. This relation has been derived by Scriven (1960), Standard (1964), Ishii (1975), Delhaye (1974) and others from integral theorem, and is sometimes called the "local instant energy jump condition".

Thus [33] and [34] confirm that [30] satisfies the total energy-balance law within each phase and at the interface. Therefore, [30] is the local instant field equation of conservation of total energy of two-phase flow.

4. SINGLE-FIELD REPRESENTATION OF THE LOCAL INSTANT CONSERVATION EQUATION

In the local instant field equations of mass, momentum and energy conservation of two-phase flow, [14], [26] and [30], one has considered two sets of density, velocity and energy fields such as $\phi_k\rho_k$, $\phi_k\mathbf{v}_k$ and $\phi_k(U_k + 0.5\mathbf{v}_k^2)$ etc. for $k = 1$ and 2. This representation enables one to formulate the local instant field equations of two-phase flow in a natural way from the basic principles of conservation of mass, momentum and total energy.

However, in view of the characteristic functions of both phases, [4] and [5], one can combine these two sets of density, velocity and energy fields into one. In view of the nature of the characteristic function ϕ_k, one introduces the following field quantities of density, velocity and energy etc.:

$$\rho = \sum_{k=1}^{2} \phi_k \rho_k, \quad \mathbf{v} = \sum_{k=1}^{2} \phi_k \mathbf{v}_k,$$

$$U = \sum_{k=1}^{2} \phi_k U_k, \quad P = \sum_{k=1}^{2} \phi_k P_k,$$

$$\tau = \sum_{k=1}^{2} \phi_k \tau_k, \quad \mathbf{F} = \sum_{k=1}^{2} \phi_k \mathbf{F}_k,$$

$$\mathbf{q} = \sum_{k=1}^{2} \phi_k \mathbf{q}_k \quad \text{and} \quad Q = \sum_{k=1}^{2} \phi_k Q_k. \qquad [35]$$

These quantities can be regarded as the field quantities because they are defined uniquely at almost every point in time and space. Using the relations satisfied by ϕ_k, i.e. [7]–[9], the products of these field quantities can be given by

$$ABC\cdots=\left(\sum_{k=1}^{2}\phi_kA_k\right)\left(\sum_{k=1}^{2}\phi_kB_k\right)\left(\sum_{k=1}^{2}\phi_kC_k\right)\cdots=\sum_{k=1}^{2}\phi_kA_kB_kC_k\ldots, \qquad [36]$$

where A, B, $C,\ldots$ represent any of the field quantities in [35].

Based on the field quantities in [35] and the relations given by [36], one can rewrite the local instant field equations of mass, momentum and total energy of two-phase flow, [14], [26] and [30]:

mass conservation

$$\frac{\partial\rho}{\partial t}+\operatorname{div}(\rho\mathbf{v})=0; \qquad [37]$$

momentum conservation

$$\frac{\partial}{\partial t}(\rho\mathbf{v})+\operatorname{div}(\rho\mathbf{v}\mathbf{v})=-\operatorname{grad}P+\operatorname{div}\tau+\rho\mathbf{F}+\mathbf{F}_sa_i; \qquad [38]$$

and

total energy conservation

$$\frac{\partial}{\partial t}[\rho(U+\tfrac{1}{2}\mathbf{v}^2)]+\operatorname{div}[\rho(U+\tfrac{1}{2}\mathbf{v}^2)\mathbf{v}]+\frac{\partial}{\partial t}(U_sa_i)+\operatorname{div}(U_sa_i\mathbf{v}_i)$$
$$=-\operatorname{div}\mathbf{q}-\operatorname{div}(P\mathbf{v})+\operatorname{div}(\tau\cdot\mathbf{v})+\rho\mathbf{F}\cdot\mathbf{v}+Q+(\mathbf{F}_s\cdot\mathbf{v}_i+\Gamma_sU_s)a_i. \qquad [39]$$

Here, differentiation is in the meaning of distribution. Because only one set of field quantities of density, velocity and energy are used here, [37]–[39] may be called the "single-field representation of the local instant field equations of two-phase flow" here.

When the interfacial force and interfacial energy terms are negligible, [37]–[39] become exactly the same as the differential equations of mass, momentum and total energy conservation in single-phase flow, except in the definition of differentiation.

In view of the above, interesting information can be deduced. In the approximation of negligible interfacial force and energy, single- and two-phase flow satisfy the same conservation equations of mass, momentum and energy. The solutions in the ordinary notion of a function represent single-phase flow and the solutions in the meaning of distribution represent two-phase flow. This conclusion gives one a clear mathematical interpretation of the thermofluid dynamics of single- and two-phase flows.

5. LOCAL INSTANT TWO-FLUID FORMATION OF TWO-PHASE FLOW

The local instant formulation of mass, momentum and energy conservation of two-phase flow, [14], [26] and [30] has another possible form. Each conservation equation can be divided into two equations, each of which includes the field quantities of only one phase.

Firstly, one considers the mass conservation equation [14]. In this equation, two sets of field quantities, i.e. $\phi_k\rho_k$ and $\phi_k\mathbf{v}_k$ for $k=1$ and 2, are used. However, is convenient to obtain the field equations which include the field quantities of only one phase, i.e. $\phi_k\rho_k$ and $\phi_k\mathbf{v}_k$ for $k=1$ or 2. In view of [10]–[25], consider the following equation:

$$\frac{\partial}{\partial t}(\phi_k\rho_k)+\operatorname{div}(\phi_k\rho_k\mathbf{v}_k)=-\rho_{ki}(\mathbf{v}_{ki}-\mathbf{v}_i)\cdot\mathbf{n}_{ki}a_i\quad(k=1,2). \qquad [40]$$

Here, again, differentiation is in the meaning of distribution.

Equation [40] is valid at any time and any location under consideration. Thus, it can be regarded as the local instant field equation of mass conservation, which includes the field quantities of only one phase. This is verified below.

Within phase k or in the region of $\phi_k = 1$, [40] can be given by

$$\frac{\partial \rho_k}{\partial t} + \operatorname{div}(\rho_k \mathbf{v}_k) = 0 \quad (k = 1, 2), \tag{41}$$

which naturally satisfies the mass conservation law within phase k.

On the other hand, in the other phase, or in the region of $\phi_k = 0$, [40] is naturally valid because both sides of [40] become zero.

Finally at the interface, in view of [10]–[13] and [22]–[24], the l.h.s. of [40] becomes

$$(\text{l.h.s. of [40]}) = -\rho_{ki}(\mathbf{v}_{ki} - \mathbf{v}_i)\cdot \mathbf{n}_{ki} a_i \quad (k = 1, 2), \tag{42}$$

which means [40] is valid at the interface. Therefore, it is clear that [40] is the local instant field equation of mass conservation of phase k. Equation [40] treats each phase separately. This treatment is similar to those adopted by Ishii (1975), Delhaye (1968), Delhaye *et al.* (1981), Bouré (1975) and Wallis (1969). However, they derived the field equations in temporal- or space-averaged forms. On the other hand, [40] is in the form of local instant conservation. Therefore, here, [40] may be called the "local instant two-fluid formulation of mass conservation of two-phase flow".

Since [40] is the divided forms of [14], when one adds the two differential equations (equations for $k = 1$ and 2 in [40]), one obtains

$$\frac{\partial}{\partial t}\left[\sum_{k=1}^{2} (\phi_k \rho_k)\right] + \operatorname{div}\left[\sum_{k=1}^{2} (\phi_k \rho_k \mathbf{v}_k)\right] = -\sum_{k=1}^{2} [\rho_{ki}(\mathbf{v}_{ki} - \mathbf{v}_i)\cdot \mathbf{n}_{ki} a_i]. \tag{43}$$

In view of [25], [43] coincides with [14] correctly.

In a similar manner, the local instant two-fluid formulation of momentum conservation can be obtained. In view of [26]–[29], this is given by

$$\begin{aligned}\frac{\partial}{\partial t}(\phi_k \rho_k \mathbf{v}_k) + \operatorname{div}(\phi_k \rho_k \mathbf{v}_k \mathbf{v}_k) &= -\operatorname{grad}(\phi_k P_k) + \operatorname{div}(\phi_k \tau_k) + \phi_k \rho_k \mathbf{F}_k \\ &\quad + \{-[\rho_{ki} \mathbf{v}_{ki}\cdot(\mathbf{v}_{ki} - \mathbf{v}_i)\mathbf{n}_{ki}] - P_{ki}\mathbf{n}_{ki} + \mathbf{n}_{ki}\cdot \tau_{ki}\} a_i \quad (k = 1, 2).\end{aligned} \tag{44}$$

Here, differentiation is in the meaning of distribution. Equation [44] is valid at any time and any location under consideration, which is confirmed in the following.

Within phase k, or in the region of $\phi_k = 1$, [44] becomes

$$\frac{\partial}{\partial t}(\rho_k \mathbf{v}_k) + \operatorname{div}(\rho_k \mathbf{v}_k \mathbf{v}_k) = -\operatorname{grad} P_k + \operatorname{div} \tau_k + \rho_k \mathbf{F}_k \quad (k = 1, 2), \tag{45}$$

which is valid within phase k. Within the other phase, or in the region of $\phi_k = 0$, [44] is naturally valid because both sides of [44] become zero.

At the interface, in view of [10]–[13] and [22]–[24], the l.h.s. of [44] reduces to

$$(\text{l.h.s. of [44]}) = -[\rho_{ki} \mathbf{v}_{ki}\cdot(\mathbf{v}_{ki} - \mathbf{v}_i)\mathbf{n}_{ki}] a_i \quad (k = 1, 2). \tag{46}$$

On the other hand, the r.h.s. of [44] becomes

$$\begin{aligned}(\text{r.h.s. of [44]}) &= (P_{ki}\mathbf{n}_{ki} - \mathbf{n}_{ki}\cdot \tau_{ki}) a_i + \{-[\rho_{ki} \mathbf{v}_{ki}\cdot(\mathbf{v}_{ki} - \mathbf{v}_i)\mathbf{n}_{ki}] \\ &\quad - P_{ki}\mathbf{n}_{ki} + \mathbf{n}_{ki}\cdot \tau_{ki}\} a_i = -[\rho_{ki} \mathbf{v}_{ki}\cdot(\mathbf{v}_{ki} - \mathbf{v}_i)\mathbf{n}_{ki}] a_i \quad (k = 1, 2).\end{aligned} \tag{47}$$

Equations [46] and [47] show that [44] is valid at the interface. Thus, [44] is the local instant two-fluid formulation of momentum conservation of two-phase flow.

Summing up the two differential equations which are obtained by putting $k = 1$ and $k = 2$ in [44], one obtains

$$\begin{aligned}\frac{\partial}{\partial t}\left[\sum_{k=1}^{2} (\phi_k \rho_k \mathbf{v}_k)\right] + \operatorname{div}\left[\sum_{k=1}^{2} (\phi_k \rho_k \mathbf{v}_k \mathbf{v}_k)\right] &= -\operatorname{grad}\left[\sum_{k=1}^{2} (\phi_k P_k)\right] \\ &\quad + \operatorname{div}\left[\sum_{k=1}^{2} (\phi_k \tau_k)\right] + \sum_{k=1}^{2} (\phi_k \rho_k \mathbf{F}_k) \\ &\quad + \sum_{k=1}^{2} \{-[\rho_{ki} \mathbf{v}_{ki}\cdot(\mathbf{v}_{ki} - \mathbf{v}_i)\mathbf{n}_{ki}] - P_{ki}\mathbf{n}_{ki} + \mathbf{n}_{ki}\cdot \tau_{ki}\} a_i.\end{aligned} \tag{48}$$

In view of [29], [48] exactly coincides with [26]. Therefore [44], the two-fluid formulation, can be regarded as the divided equations of [26].

Finally, the local instant two-fluid formulation of total energy conservation is considered. In view of [30]–[34], this is given by

$$\begin{aligned}
&\frac{\partial}{\partial t}[\phi_k\rho_k(U_k+\tfrac{1}{2}\mathbf{v}_k^2)]+\operatorname{div}[\phi_k\rho_k(U_k+\tfrac{1}{2}\mathbf{v}_k^2)\mathbf{v}_k]\\
&\quad=-\operatorname{div}(\phi_k\mathbf{q}_k)-\operatorname{div}(\phi_k P_k\mathbf{v}_k)+\operatorname{div}(\phi_k\tau_k\cdot\mathbf{v}_k)+\phi_k\rho_k\mathbf{F}_k\cdot\mathbf{v}_k+\phi_k Q_k\\
&\qquad+[-\rho_{ki}(U_{ki}+\tfrac{1}{2}\mathbf{v}_{ki}^2)(\mathbf{v}_{ki}-\mathbf{v}_{\mathrm{i}})\cdot\mathbf{n}_{ki}-\mathbf{n}_{ki}\cdot\mathbf{q}_{ki}-P_{ki}\mathbf{v}_{ki}\cdot\mathbf{n}_{ki}+\mathbf{n}_{ki}\cdot(\tau_{ki}\cdot\mathbf{v}_{ki})]a_{\mathrm{i}}\\
&\qquad\qquad(k=1,2).\qquad[49]
\end{aligned}$$

Here, again, differentiation is in the meaning of distribution. Equation [49] is valid at any time and any location under consideration, which is verified in the following.

Within phase k, or in the region of $\phi_k = 1$, [49] becomes

$$\begin{aligned}
&\frac{\partial}{\partial t}[\rho_k(U_k+\tfrac{1}{2}\mathbf{v}_k^2)]+\operatorname{div}[\rho_k(U_k+\tfrac{1}{2}\mathbf{v}_k^2)\mathbf{v}_k]\\
&\qquad=-\operatorname{div}\mathbf{q}_k-\operatorname{div}(P_k\mathbf{v}_k)+\operatorname{div}(\tau_k\cdot\mathbf{v}_k)+\rho_k\mathbf{F}_k\cdot\mathbf{v}_k+Q_k\quad(k=1,2),\qquad[50]
\end{aligned}$$

which is naturally valid within phase k. One the other hand, within the other phase, or in the region of $\phi_k = 0$, [49] is naturally valid because both sides of [49] reduce to zero.

At the interface, in view of [10]–[13] and [22]–[24], the l.h.s. of [49] becomes

$$(\text{l.h.s. of [49]})=-[\rho_{ki}(U_{ki}+\tfrac{1}{2}\mathbf{v}_{ki}^2)(\mathbf{v}_{ki}-\mathbf{v}_{\mathrm{i}})\cdot\mathbf{n}_{ki}]a_{\mathrm{i}}\quad(k=1,2).\qquad[51]$$

On the other hand, the r.h.s. of [49] becomes

$$\begin{aligned}
&(\text{r.h.s. of [49]})=\\
&[\mathbf{n}_{ki}\cdot\mathbf{q}_{ki}+P_{ki}\mathbf{v}_{ki}\cdot\mathbf{n}_{ki}-\mathbf{n}_{ki}\cdot(\tau_{ki}\cdot\mathbf{v}_{ki})]a_{\mathrm{i}}\\
&\quad+[-\rho_{ki}(U_{ki}+\tfrac{1}{2}\mathbf{v}_{ki}^2)(\mathbf{v}_{ki}-\mathbf{v}_{\mathrm{i}})\cdot\mathbf{n}_{ki}-\mathbf{n}_{ki}\cdot\mathbf{q}_{ki}-P_{ki}\mathbf{v}_{ki}\cdot\mathbf{n}_{ki}\\
&\quad+\mathbf{n}_{ki}\cdot(\tau_{ki}\cdot\mathbf{v}_{ki})]a_{\mathrm{i}}=-[\rho_{ki}(U_{ki}+\tfrac{1}{2}\mathbf{v}_{ki}^2)(\mathbf{v}_{ki}-\mathbf{v}_{\mathrm{i}})\cdot\mathbf{n}_{ki}]a_{\mathrm{i}}\quad(k=1,2).\qquad[52]
\end{aligned}$$

Equations [51] and [52] mean [49] is valid at the interface. Thus, [49] is the local instant two-fluid formulation of total energy conservation.

When one takes the summation of the two equations with $k = 1$ and $k = 2$ in [49], one obtains

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{\sum_{k=1}^{2}[\phi_k\rho_k(U_k+\tfrac{1}{2}\mathbf{v}_k^2)]\right\}+\operatorname{div}\left\{\sum_{k=1}^{2}[\phi_k\rho_k(U_k+\tfrac{1}{2}\mathbf{v}_k^2)\mathbf{v}_k]\right\}\\
&\quad=-\operatorname{div}\left[\sum_{k=1}^{2}(\phi_k\mathbf{q}_k)\right]-\operatorname{div}\left[\sum_{k=1}^{2}(\phi_k P_k\mathbf{v}_k)\right]+\operatorname{div}\left[\sum_{k=1}^{2}(\phi_k\tau_k\cdot\mathbf{v}_k)\right]\\
&\qquad+\sum_{k=1}^{2}(\phi_k\rho_k\mathbf{F}_k\cdot\mathbf{v}_k)+\sum_{k=1}^{2}(\phi_k Q_k)\\
&\qquad+\sum_{k=1}^{2}[-\rho_{ki}(U_{ki}+\tfrac{1}{2}\mathbf{v}_{ki}^2)(\mathbf{v}_{ki}-\mathbf{v}_{\mathrm{i}})\cdot\mathbf{n}_{ki}-\mathbf{n}_{ki}\cdot\mathbf{q}_{ki}-P_{ki}\mathbf{v}_{ki}\cdot\mathbf{n}_{ki}+\mathbf{n}_{ki}(\tau_{ki}\cdot\mathbf{v}_{ki})]a_{\mathrm{i}}.\qquad[53]
\end{aligned}$$

In view of [32] and [34], [53] exactly coincides with [30]. Therefore, [49] can be regarded as the divided equations of [30].

Thus, the local instant two-fluid formulation of mass, momentum and energy conservation equations, [40], [44] and [49], have been obtained. This formulation consists of two sets of local instant field equations of mass, momentum and total energy conservation of both phases. In other words, there are six differential equations which determine the density, velocity and energy fields of both phases. As shown in [40], [44] and [49], any one of these six equations has the field quantities concerning only on phase. However, these six equations are dependent on each other through the interfacial terms.

As shown in [40], [44] and [49], the interfacial transfer terms appear as the source terms at the interface. Therefore they are given in terms of the local instant interfacial area concentration a_i, which is represented by distribution. As shown in [40], the interfacial mass transfer terms are given by

$$-\rho_{ki}(\mathbf{v}_{ki}-\mathbf{v}_i)\cdot\mathbf{n}_{ki}a_i \quad (k=1,2).$$

They are related to each other in the interfacial mass balance, given by [25].

As shown in [44], the local instant interfacial momentum transfer terms are given by

$$\{-[\rho_{ki}\mathbf{v}_{ki}\cdot(\mathbf{v}_{ki}-\mathbf{v}_i)\mathbf{n}_{ki}]-P_{ki}\mathbf{n}_{ki}+\mathbf{n}_{ki}\cdot\tau_{ki}\}a_i \quad (k=1,2).$$

These terms are related to each other through the interfacial momentum balance, given by [29].

In view of [49], the local instant energy transfer terms are given by

$$[-\rho_{ki}(U_{ki}+\tfrac{1}{2}\mathbf{v}_{ki}^2)(\mathbf{v}_{ki}-\mathbf{v}_i)\cdot\mathbf{n}_{ki}-\mathbf{n}_{ki}\cdot\mathbf{q}_{ki}-P_{ki}\mathbf{v}_{ki}\cdot\mathbf{n}_{ki}+\mathbf{n}_{ki}\cdot(\tau_{ki}\cdot\mathbf{v}_{ki})]a_i \quad (k=1,2).$$

These terms satisfy the relation given by [34].

Furthermore, the six equations in the two-fluid formulation include the characteristic function of each phase, $\phi_k(k=1,2)$. The characteristic function puts further constraints on the two-fluid formulation. In view of [4]–[6], these constraints are given by

$$\sum_{k=1}^{2}\phi_k=1 \qquad [54]$$

and

$$\phi_k\phi_j=0\,(k\neq j). \qquad [55]$$

Now the complete description of the local instant two-fluid formulation of the mass, momentum and total energy conservation of two-phase flow has been obtained. This formulation is composed of the six conservation equations of mass, momentum and total energy ([40], [44] and [49]) and the constraint equations for the interfacial transfer terms ([25], [29] and [34]) and the characteristic functions ([54] and [55]).

6. CONCLUSIONS

The local instant field equations of two-phase flow have been formulated by introducing the notion of distribution which is an extended notion of a function. The difficulties in differentiation of the discontinuous function and in representation of the source terms at the interface, which appear in the local instant formulation, have been overcome by using distribution.

The physical parameters of each phase, such as density, velocity and energy etc., have been expressed as the field quantities in terms of the characteristic functions of both phases. The local instant volume fractions are conveniently defined by these characteristic functions. In addition to these parameters, the field quantities related to the interface have been introduced in terms of the local instant interfacial area concentration.

Based on the field quantities of two-phase flow thus defined, the local instant field equations of mass, momentum and energy conservation in two-phase flow have been rigorously formulated. The mass conservation equation is given by [14], the momentum conservation equation by [26] and the total energy conservation equation by [30]. In the momentum and total energy conservation equations, the interfacial force and the interfacial energy terms appear as source terms at the interface.

The two sets of field quantities, which have been defined for both phases, have been combined into one set. The newly defined set of field quantities has simplified the local instant field equations described above and gives the single-field representation of the local instant conservation equations of mass, momentum and total energy which are given by [37]–[39], respectively.

In the approximation of the negligible interfacial force and energy, the conservation equations of this representation coincide with the conservation equations of single-phase flow, except in the definition of differentiation. This indicates that the field quantities of single- and two-phase flow can be given as the solutions of the same conservation equations of mass, momentum and energy. Single-phase flow corresponds to the solution in the meaning of the ordinary notion of a function, whereas two-phase flow corresponds to the solutions in the meaning of distribution.

The local instant two-fluid formulation of two-phase flow has also been derived. Two sets of mass, momentum and total energy conservation equations ([40], [44] and [49]) have been derived. Each of these six equations has the field quantities concerning only one phase. However, these equations are dependent on each other through the interfacial transfer terms. The constraint equation for the interfacial transfer of mass is given by [25], that of momentum by [29] and that for total energy by [34]. Furthermore, the constraint equations for the characteristic functions are given by [54] and [55].

The local instant formulation derived here is one of the most rigorous formulations of two-phase flow. When the field equations in this formulation are solved under appropriate initial and boundary conditions, the behavior of two-phase flow can be predicted very accurately. In particular, in analyzing the microscopic characteristics of two-phase flow, such as turbulence etc., this local instant formulation is indispensable.

Furthermore, this formulation constitutes a useful tool in evaluating the applicability of the various simplified models of two-phase flow.

NOMENCLATURE

$a_i(x, y, z, t)$ = Local instant interfacial area concentration
A_k, B_k, C_k = Physical parameters of phase k ($k = 1, 2$)
$f(x, y, z, t)$ = Function representing the interface
$\mathbf{F}$ = External force per unit mass, defined by [36]
$\mathbf{F}_k$ = External force per unit mass of phase k ($k = 1, 2$)
$\mathbf{F}_s$ = Interfacial force per unit interfacial area
$h(w)$ = Heaviside function
$\mathbf{n}_{ki}$ = Outward unit normal vector of phase k ($k = 1, 2$)
P = Pressure, defined by [36]
P_k = Pressure of phase k ($k = 1, 2$)
P_{ki} = Pressure of phase k at the interface ($k = 1, 2$)
$\mathbf{q}$ = Heat flux vector, defined by [46]
$\mathbf{q}_k$ = Heat flux vector of phase k ($k = 1, 2$)
$\mathbf{q}_{ki}$ = Heat flux vector of phase k at the interface ($k = 1, 2$)
Q = Heat generation rate per unit volume, defined by [36]
Q_k = Heat generation rate per unit volume of phase k ($k = 1, 2$)
t = Time
$T(w)$ = Distribution
U = Internal energy per unit mass, defined by [36]
U_k = Internal energy per unit mass of phase k ($k = 1, 2$)
U_{ki} = Internal energy per unit mass of phase k at the interface ($k = 1, 2$)
U_s = Interfacial energy per unit interfacial area
$\mathbf{v}$ = Velocity, defined by [36]
$\mathbf{v}_i$ = Velocity of the interface
$\mathbf{v}_k$ = Velocity of phase k ($k = 1, 2$)
$\mathbf{v}_{ki}$ = Velocity of phase k at the interface ($k = 1, 2$)
v_{kx}, v_{ky}, v_{kz} = x, y-, and z-components of the velocity of phase k ($k = 1, 2$)
w = Variable
x, y, z = Space coordinates

Greek symbols

Γ_s = Generation rate of interfacial area concentration per unit interfacial area

$\delta(w)$ = The δ-function

ρ = Density, defined by [36]

ρ_k = Density of phase k ($k = 1, 2$)

ρ_{ki} = Density of phase k at the interface ($k = 1, 2$)

τ = Stress tensor, defined by [36]

τ_k = Stress tensor of phase k ($k = 1, 2$)

τ_{ki} = Stress tensor of phase k at the interface ($k = 1, 2$)

$\phi_k(x, y, z, t)$ = Characteristic function of phase k ($k = 1, 2$)

$\psi(w)$ = Arbitrary function

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